Theory-Informed Machine Learning for Grand Strategic Decision-Making: AI-Driven Hierarchical Framework

Caesar Wu, Rajkumar Buyya, and Pascal Bouvry

Abstract Contemporary grand strategists, political scientists, and economists have formulated various theoretical models to analyse historical events and forecast or predict future scenarios for strategic decision-makers. However, many of these models remain predominantly qualitative and narrative-led. While they provide some valuable principles and domain knowledge, they often lack empirical quantification, which limits their applications for making precise decisions over time. Traditional frameworks usually struggle to address issues such as interconnectivity, dynamics, nonlinearity, feedback, emergencies, co-evolution, unpredictability, uncertainty, and ambiguity. To address this gap, we propose a hierarchical framework underpinned by seven decision layers, which can be quantified by Theory-Informed Machine Learning (TIML) methods. This framework enables us to manage various grand strategies or strategic challenges. We argue that a grand strategy is an abstract pattern of human intelligence that emerges from multiple decision layers below and is driven by emotional rewards from above. It transcends the way of balancing means with the end and context. We consider that a grand strategy is a type of strange attractor, which is a deterministic chaos. Determinism implies that our time, resources, capability, and cultural background are bounded. Chaos entails unpredictable long-term consequences. We aim to create a new computational model that can craft a robust grand strategy driven by TIML based on chaos and complexity theories.

C. Wu (✉) · P. Bouvry
The University of Luxembourg, Faculty of Science, Technology and Medicine, Esch-sur-Alzette, Luxembourg
e-mail: Caesar.Wu@Uni.lu

R. Buyya
The University of Melbourne, School of Computing and Information Systems, Melbourne, Australia

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1 Introduction

The meaning of grand strategy dates back to the ancient military thinker, Sun Tzu's The Art of War and the ancient historian, Thucydides' The Peloponnesian War, in the fifth century BCE. However, the modern notion of grand strategy was first coined by French General Andre Beaufre and French sociologist Raymond Aron. Beaufre emphasized "total strategy," while Aron mixed the term with policy and strategy [1]. The word "grand" comes from the old French "grant" or "grand", meaning "large, great, or important." The word itself derives from the Latin "grandis," which means "great, big, or lofty."

Traditionally, a technical definition of strategy means "art of war," "science of war," or "art of the game" [1]. We often use "grand strategy" and "strategy" interchangeably. It depends on the context. However, the notion of grand strategy should be one level higher than strategy in terms of scale, scope, complexity, objectives, measurements, and timeframe. B. H. Liddell Hart cast the British version of the definition. He argued that strategy is "the art of distributing and applying military means to fulfil the ends of policy." [2] In everyday conversation, the conventional meaning of strategy implies a deliberate plan, prioritizing decisions, and competitive tactics. It might reduce to a good idea without the necessary underlying thought or development [36]. However, Mintzberg [3] rejected the idea that a strategy is the consequence of planning. He argued that strategy is an emergent phenomenon that grows from a broad and complex environment.

In contrast to a narrow perspective, Gaddis [4] offers a coherent and abstractive framework for defining a grand strategy based on the liberal arts approach, which integrates literature, philosophy, and history. He describes the grand strategy as balancing "end" (goals) with "means" (resources) and context to shape long-term success.

In the past, many prestigious scholars [2–5], theorists [7, 10, 12], and strategists [8, 9] framed grand strategy or strategy from different (business, military, diplomatic, historical, policy, political, cultural, philosophical, evolutionary, geopolitics, and even emotional) perspectives to build theoretical principles that can predict future scenarios based on historical events. However, some models remain predominantly analytical in nature. While these guiding principles offer practical guidelines to many decision-makers, they often lack empirical quantification and fail to address complex and highly dynamic situations. The most critical issue is that many theories are overly dogmatic and exhibit cultural biases.

In other words, traditional frameworks often struggle to cope with the complexities of modern strategic environments and context, which involve interconnections, dynamics, nonlinearity, complexity, feedback, unpredictability, ambiguity,

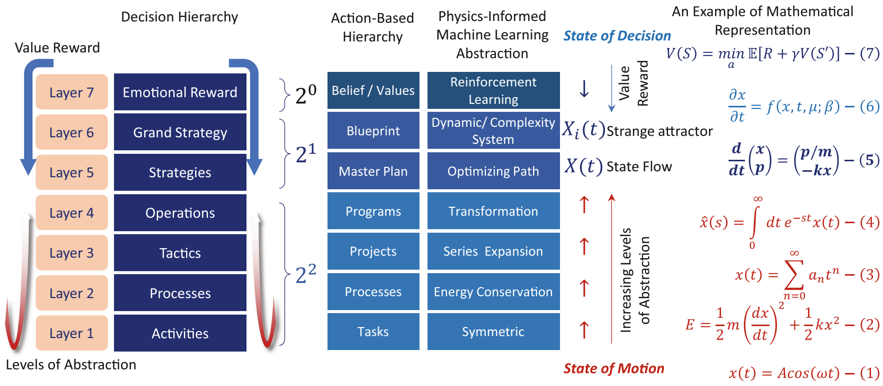

Fig. 1 Seven layers of the hierarchical framework

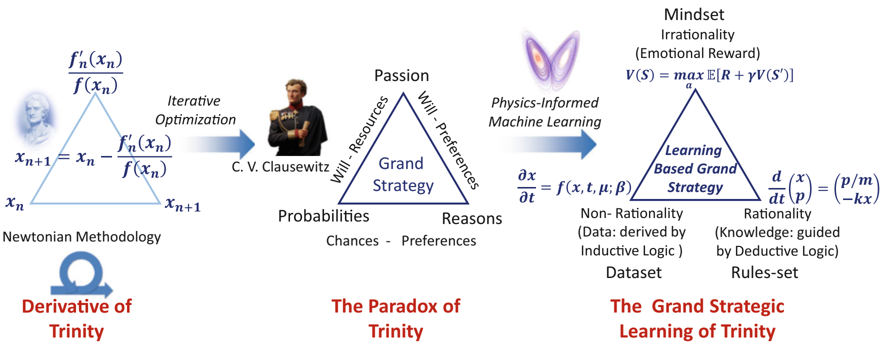

Fig. 2 From the derivative of trinity to the learning of trinity

and emergencies [13]. To address this gap, we propose a hierarchical framework underpinned by seven decision layers, which can be quantified by Artificial Intelligence(AI) / Machine Learning (ML) methods based on three types of learning: deep learning, reinforcement learning, and rule-based learning (See Fig. 1). Figure 1 illustrates a decision hierarchy that has seven layers, from the bottom up: activities, processes, tactics, operations, strategies, grand strategy, and emotional reward (left of Fig. 1). The essence of the proposed framework can be considered as an isomorphism of action-based hierarchy and Physical-Informed Machine Learning (PIML) [15, 16] abstraction. We can use various mathematical representations to describe a state's abstraction from a state of motion to a state of decision. There are three learning directions: value reward via reinforcement learning (RL) from the top down, and levels of abstraction via "deep learning" (DL) from bottom up based on data. TIML is hovering around a phase (policy) space in the middle based on the theory of knowledge. We call it the learning trinity, which is derived from Clausewitz's paradox of trinity and Newton's derivative trinity. (See Fig. 2).

1.1 Research Problem and Assumptions

The proposed hierarchical framework addresses the research problem: how to create a coherent model for a robust grand strategy that can predict unpredictability in a highly dynamic, complex, chaotic, ambiguous, and evolutionary environment and ensure long-term prosperity. The idea of grand strategy often refers to a policy from a military, geopolitical, and political science perspective. This study, however, primarily focuses on a business problem from a long-term evolutionary perspective. A typical example is the board directors of a telecom firm who repeatedly want to decide how to roll out the next generation of networks over decades continuously. If a strategy is to solve the issue of rolling out one generation of a network, then a grand strategy is to tackle the problem of deploying many generations of networks. Of course, the model can also be applied to other fields. To some extent, the problem is a part of managerial science or operational research.

1.2 Contributions

Our primary contributions are as follows:

1. Generate an innovative hierarchical framework that emerged from seven elementary layers (Refer to Fig. 1), which are divided into three digital states: levels of abstraction (state of motion or manipulation), emergent (policy state), and value rewards (decision state). These blocks correspond to three types of learning methods (reinforcement, deep, and rule-based or theory-informed learning). This framework is a departure from the traditional approach of crafting a grand strategy that often hovers at the level of impractical and intangible guidelines.
2. We propose an integrated learning method, related to Theory-informed Machine Learning (TIML), to control an actuation variable for achieving the desired grand strategic outcome.
3. We demonstrate how to generate a robust grand strategy by formulating a strange attractor in policy space by defining the number of variables and a set of parameters. We argue that if the formulated grand strategy can remain on or near the strange attractor, it can efficiently cooperate with all complex, nonlinear, dynamic, ambiguous, and unpredictable issues over a long time period.
4. By applying chaos and complexity theory, we formulated the grand strategy (emergent state), which is a desired strange attractor (a momentum relationship or pattern of defined variables) in a policy (phase) space.
5. We also demonstrated how to harness complexity and chaos theory through a practical example by establishing three ordinary differential equations (ODE) based on defining three variables: economic growth (E) (i.e. sales revenue), population size (P) (i.e. full-time equivalent employees), and competitiveness(C) (i.e. new features or products released to the market).

1.3 Outline of the Paper

The rest of the paper will be organized as follows: Section 2 is the literature review of previous works regarding the recent development of strategy, grand strategy, dynamic system, chaos theory, complex system, physics-informed machine learning, and other computational methods. Section 3 describes the basic principle to establish the hierarchical framework and its underlying logic. Section 4 highlights the model's assumptions and the experiment setup. Section 5 is the experimental results and details of the analysis of the results. Section 6 is the conclusion and future work.

2 Literature Review

The notion of strategy and grand strategy has become popularized in both academic and industrial domains. Many contemporary scholars and experts have provided various theories, frameworks, principles, and guidelines for strategic decision-makers. However, many scholars treated this subject qualitatively. There has been a lack of quantitative analysis on this topic. Betts [17] criticized the concept of grand strategy as overly abstract and often disconnected from practical realities. His central argument revolves around the tension between the theoretical utility of grand strategy and its limitations in real-world applications due to unpredictable variables. The key themes of his critique derive from five perspectives: definitional ambiguity, grandiosity (exaggeration), the gap between principles and practice, flexibility versus rigid planning, and relevance in contemporary practices.

Mintzberg [18] also challenged the conventional framework of traditional grand strategy and emphasize adaptability. He defined real strategy as a pattern in a stream of actions arising from the interplay between intentional plans and emergent strategy (states). By integrating decades of research, Mintzberg's work remains foundational for understanding grand strategy as a living, adaptive process rather than a static plan. However, he failed to identify the details of a stream of actions.

Similarly, Milevski [19] echoes Bett's view from an evolutionary perspective. He argued that no standardized definition for strategy or grand strategy existed. People adopted it for their own purposes. There is a theoretical vacuum regarding grand strategic theory. To fill this theoretical vacuum, Gray [20] proposed four types of theories: (1) Politics and strategy, (2) Chaos and order, (3) Understanding complexity, (4) Coherent theory. Schelling [21] pioneered the mathematical modelling of strategic interaction. His basic concept of the game theory model is a state of equilibrium, iteration, and signalling, which does not apply to a chaotic, nonlinear, and dynamic system whose state evolves over time, usually in response to inputs, internal rules, and feedback loops. Boyd [22] created a conceptual model of an "observe-orientation-decision-action" (OODA) loop for strategic manoeuvring in a grand strategy context. His model was deeply inspired by thermodynamics and systems theory.

Goodwin and Punzo [32] introduced the nonlinear dynamics and chaotic ideas in economic modelling. They developed a nonlinear business model that showed limited cycles and complex oscillations, early steps toward chaos. Stacey [33, 34] established chaos and complexity theory for strategy and organizational dynamics. He popularized the idea of "the edge of chaos" in business contexts. Lissack [35] also promoted chaos and complexity concepts for leadership and organisational behaviour, which influence strategic thinking.

However, no scholar has formally articulated a comprehensive framework based on complexity or chaos theories to craft a grand strategy which uses mathematical models of nonlinear dynamics for strategic thinking. Nevertheless, some authors [23, 24] noticed that the essential characteristics of strategic thinking are nonlinearity, feedback, chaos, ambiguity, and unpredictability. Simpson [25] promoted the idea of "from the ground up" or emergent patterns of a grand strategy. With the rise of recurrent neural networks (RNN) [26] Raissi et al. [27] introduced a physics-informed machine learning (PIML) method to boost efficiency for the training model. The essence of PIML is data-driven learning combined with physical laws, such as conservation laws, Newtonian laws, and differential equations, for chaotic, complex, and multiscale systems. PIML has become an indispensable tool for computing grand strategy. The PIML method inspires the TIML [37] approach that fosters a synergy between theory, knowledge and data.

3 Hierarchical Frameworks

Based on Mintzberg's idea that a grand strategy is a pattern in a stream of actions arising from the interplay between emergent states, we can model the stream of actions as supporting layers of the grand strategy from a hierarchical perspective. To create a comprehensive framework for a grand strategy, we need more than the strategic element because we want any grand strategy to be underpinned by real activities. It avoids disconnection from reality.

To quantify a grand strategy from the bottom up, we define a seven-layer hierarchy that is inspired by the Open System Interconnection (OSI) architecture. The logic behind the seven-layer architecture is to create upward abstraction and emergence, as well as downward value-driven illumination. This architecture enables a comprehensive coverage of all functional aspects of a grand strategy. Furthermore, we can translate the seven layers into a learning framework for trinity, for practicality (Refer to Fig. 2). The bottom four layers represent content that reflects the logical aspects of grand strategy, similar to the OSI model's data link layer. The top three are agent layers influenced by a value function or subjective belief, equivalent to hosting layers of the OSI model. The goal of this framework is to enable predictability for decision-makers in a chaotic and dynamic environment.

The backbone of crafting a grand strategy is time series forecasting, governed by the state layers in policy space. We will employ various time series forecasting techniques, including both shallow learning and deep learning methods, to extract patterns from data generated by operations, tactics, processes, and activities. However, the top layer of emotional reward (shared value and belief) also influences policy space. The essence of the top layer depends on the decision-makers' culture formalization [11] or background, even geographic location, because culture is reality and "Geography is a destiny". The phrases underscore the immutable force shaping our way of thinking, and cultural transformation unfolds over the course of centuries [6]. Peter Drucker later captured a parallel wisdom, indicating that "Culture eats strategy for breakfast."

4 Model Definition, Assumptions, and Experiment Setup

To test the above framework (hypothesis), we defined three variables: (1) economic capacity (E), which is a state of economics that is roughly equivalent to measuring a company's sales revenue. (2) Population size (P) implies the state of the workforce or the number of full-time equivalent (FTE) employees. (3) Competitiveness (C) means a state of competitive advantage [28] or some new products released onto the market. It is fueled by economic capacity (the average research funding) and may be dampened by population size in terms of employment costs. One of the measurements is the average sales revenue per FTE. In addition, we also have the actuation: μ to control the desired result and a set of parameters (α, β, γ, δ, ϵ, ϑ, η) to model the grand strategy of business development. Based on defined variables and a set of parameters, we can establish the following three ordinary differential equations (ODEs).

$$\frac{dE}{dt} = \alpha E - \beta P \bullet C - \mu \quad (1)$$

$$\frac{dP}{dt} = \gamma P \left(1 - \frac{P}{k}\right) + \delta E \bullet C \quad (2)$$

$$\frac{dC}{dt} = \vartheta + \epsilon C \left(E - \eta\right) \quad (3)$$

Where α: intrinsic economic growth rate. β: sensitivity of the economy to population/competitiveness pressure. γ: population growth rate. K: carrying capacity (max sustainable population). δ: economic stress impact on population. ϵ: competitiveness growth. θ: population-driven drag on competitiveness. According to Fig. 1, the decision hierarchy of four layers from the bottom (activities, processes, tactics, and operations) corresponds to four levels of action hierarchy (tasks, processes, projects, and programs). We assume that these components contribute to economic growth and competitiveness. Like Lorenz's system [29], the decision hierarchy has its convention (abstraction) "motion."

Three specified ordinary differential equations (ODEs) are also inspired by a combination of the effects of Lorenz, Rossler, [30] and Chen-Lee's [31] (See Table 1). The first equation implies that the economic state depends on its current state and nonlinear effects from the workforce (FTE) size and competitiveness. The second equation indicates that the workforce's state relies on a nonlinear state of population, plus a nonlinear component of economics, multiplied by competitiveness. The third equation articulated the relationship between the competitiveness state, the economic element, and the workforce state.

If we fine-tune the initial conditions and specified parameters, we can achieve a strange attractor where a system's behavior over time is drawn towards a particular set of states, which forms a shape in policy (or phase) space. The characteristic of the system is that it never repeats exactly, but also never becomes random. It is constrained, but still unpredictable. The system formulates a learning based grand strategy. It is inspired by Newton's derivative of the trinity and Clausewitz's paradox of the trinity (See Fig. 2) [14].

Overall, the strategic learning is driven by three types of computations: (1) The non-rationality (probability computations). (2) The irrationality is based on a reward function (current position or state and value rewards). (3) The rationality can be determined by the Hamilton equation, which can be abstracted from the bottom.

5 Experiment Results and Analysis

The experiment is to specify a set of parameters with the initial conditions so that the EPC system can form a strange attractor in phase (or policy) space (Refer to Fig. 3). The meaning of a strange attractor is that the decision-maker (strategist) can forecast the momentum and position for the EPC system in policy space. It provides powerful insight regarding the relationship of the specified variables. It is a pattern in a chaotic system in which the system's behavior over time is drawn toward a particular set of states that form a shape in phase space, like the "gravitational pull" in the phase space. It suggests that chaos has some kind of hidden order, although the chaotic system is unpredictable in detail.

We argue that the EP's strange attractor or system can overcome many challenges when a grand strategy is crafted, including nonlinearity, interdependence, co-evolution, feedback loops, emergence, ambiguity, and unpredictability. In the context of grand strategy, strange attractors offer decision-makers different levels of abstraction and insight in policy space, enabling them to execute a robust grand strategy with ease.

The practical interpretation of the EPC's long-term economic consequences (gain and loss of a company's sales revenue) is the abstraction of the content layers (activities, processes, tactics, and operations). Sales revenues (or economic consequences) could fluctuate over time in the time domain. The grand strategy should remain in the "attracted" position in the policy space. Conversely, if the formulated grand strategy cannot stay in the orbit of the trajectory of a strange

Table 1 Comparison of Lorenz, Rossler, Chen-Lee, and EPC strange attractors

Aspects	Lorenz (1963)	Rossler (1976)	Chen-Lee (2008)	EPC (20025)
Nonlinear	xy, xz	xz	xy, xz, yz	CP, P^2, CE
Symmetry	$x \to -x,\ y \to -y$	No symmetry	No symmetry	No symmetry
Attractor shape	Butterfly (two lobes)	Single spiral	Toroidal	Loop
Physical origin	Atmospheric convection	Abstract chaos model	Rigid body rotation	Level of abstraction
Parameter roles	σ, ρ, β(fluid dynamics)	a, b, c (abstract tuning)	a, b, c(damping/energy terms)	$\alpha, \beta, \gamma, \delta, \epsilon, \vartheta, \eta$
Complexity	High (strong coupling)	Low (simple structure)	Moderate (multiple nonlinearities)	Low
Parameter example for chaos	$\sigma = 10$, $\rho = 28$, $\beta = 8/3$	$a = b = 0.2$, $c = 5.7$	$a = 5$, $b = -10$, $c = -3.8$	$\alpha, \beta, \gamma, \delta, \epsilon, \vartheta, \eta$
Applications	Fluid dynamics	Biology/chemistry	Security communication, circuit design	Grand strategic decision

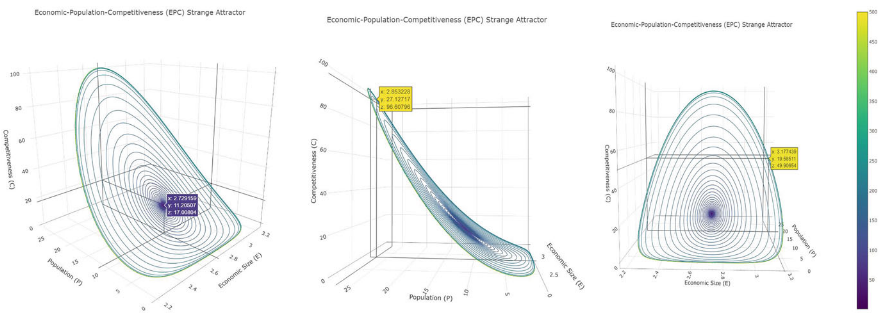

Fig. 3 The EPC strange attractor

attractor, the long-term grand strategy may be hard to implement. This also means that the grand strategy should maintain the proper balance of policy momentum and position, rather than responding to immediate needs. We argue that the EPC system reflects a relationship among three specified variables with three defined ODEs, and these ODEs demonstrate the following characteristics:

- The EPC system follows defined rules. It means that the EPC system is deterministic, but the consequences of these interactive variables are unpredictable.
- Once certain initial conditions and defined parameters are given, the system can form a strange attractor. We can predict a state in phase (policy) space over time. However, we cannot predict the exact result (i.e. sales revenue) in the time domain (a particular year).
- The strange attractor is sensitive to initial conditions, and a slight difference will grow over time.
- The EPC system does not spiral off to infinity. It stays within a particular region.
- It never cycles exactly but stays "attracted" to a specific region.
- It demonstrated a fractal structure, which we can see complexity within complexity if we zoom in.
- It produces unpredictable and complex behavior.

Generally, we can craft a grand strategy in various states between order and disorder, such as static, equilibrium, ergodic, metastable, edge of chaos, self-organized criticality, strange attractor, and random. Traditional forecasting methods often assume that a decision environment is static [22] and the equilibrium [21] state. However, when the decision environment becomes more complex and chaotic among multiple players, these traditional theories cannot cope with it. Therefore, we propose this hierarchical framework to formulate a grand strategy that can work with chaotic states and unpredictability.

The experiment also demonstrates that if we want to control the policy momentum and position, we can justify the variable μ for the desired outcome. In other words, we can harness chaos and complexity theories to control the strategic decisions.

6 Conclusions and Future Works

Traditional methods of grand strategy are challenging to apply in a chaotic, nonlinear, dynamic, complex, emergent, and interactive decision environment, as they often assume the decision environment is either static or in equilibrium. Many strategic theories offer only a few guiding principles. Although these theories are sometimes helpful, they lack empirical quantification and cannot provide precise guidance for implementing a grand strategy. Therefore, developing a new framework to address this gap is necessary. This study proposes a novel framework based on the TIML method to address the "unpredictable" problem for a grand strategy.

This new framework is supported by seven decision layers: activities, processes, tactics, operations, strategies, grand strategy, and emotional reward. These seven layers can be further divided into three learning blocks: the non-rational block (dataset), the rational block (rule-set), and the irrational block (mindset). The non-rational block can handle abstract and emergent issues. The rational block copes with momentum and position, which is often dynamic, complex, nonlinear, and chaotic. The irrational block manages the value problems. This framework enables us to leverage reinforcement learning (RL) driven from the top layer, time-series forecasting (i.e. Long Short-Term Memory: LSTM, Gated Recurrent Unit: GRU, Transformer-based neural networks) emerges from the bottom up (based on the real datasets), and TIML produces the targeted strange attractor for the desired outcome in the middle.

The control variable μ can adjust the policy momentum and position if the result of time series forecasting deviates from a desired strange attractor. If we assume the specified strange attractor is a robust grand strategy, it demonstrates a hidden order in the policy space. This means that if we can harness this hidden order, we can implement a grand strategy well. The future work will be on how to evaluate and change the variable μ based on the results of time series forecasting.

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